pair of equilibria "from air." They can form from an unstable equilibrium as a result of bifurcation of the limit cycle. Finally, they can also occur in the case of bilateral bifurcation, associated with an exchange of stability between two equilibria – one inside the invariant subspace and one outside it.

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USE OF A THREE-COMPONENT MODEL TO COMPUTE GAS SUSPENSION FLOW AND RAREFIED FLOW OVER BODIES

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A 4-component model to describe flow of a suspension (gas with solid particles) over bodies is proposed in [1]. The suspension is a mixture of four components: carrier gas and three kinds of particles, which do not collide with incident s particles, orderly moving reflected r particles, and randomly moving t particles. It is postulated that any two colliding particles (only pair collisions are considered) occur in type t. The particles are assumed to be identical spheres whose diameter d_0 is much less than the characteristic body dimension, while the density ρ_0 is much larger than that of the gas. The velocity distribution of the t particles is assumed to be nearly Maxwellian, and for the t component we use certain results of kinetic theory obtained for a gas consisting of spherical molecules. Here we neglect the influence of resistance of the carrier gas and the possible inelasticity of collisions on the form of the formulas for flux of mass, momentum, and energy. These factors are accounted for in computing the kinetic energy of random motion of particles U_t, determined from the balance equation, which has terms describing dissipation of this energy due to the above causes.

The hypotheses listed, described in detail in [1], lack a rigorous basis, but with them we can construct a rather simple suspension model accounting for random motion of particles, and correctly describing the screening influence of reflected particles, as shown by comparing the computations of [2] with experimental data [3].

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The practical form of this model is very complex. However, in some cases there is no gain in using the model in the full volume, and the random motion of particles can be accounted for in simple models of the medium. For example, [4] solved the problem of flow over a sphere, the fluid being a suspension based on a 3-component model, where the r and t components were combined. This condition holds, for example, when the body surface has roughness comparable with particle dimensions.

In this paper we consider the case where one neglects the influence of the gas on the motion. These conditions are achieved in experiments where the velocity relaxation length of the particles is much larger than the characteristic body length [3, 5], and the flow can be considered as flow of solid particles (without allowing for the carrier). If we assume that the gas of t particles is inviscid and does not conduct heat, then the balance equations can be written in the following form:

$$\partial\rho_i/\partial t + \mathrm{div}\,(\rho_i \mathbf{v}_i) = J_i \ (i = s,\ r,\ t);$$
$$\rho_t \partial \mathbf{v}_t/\partial t + \rho_t(\mathbf{v}_t \cdot \nabla)\,\mathbf{v}_t = -\nabla p_t - J_s(\mathbf{v}_s - \mathbf{v}_t) - J_r(\mathbf{v}_r - \mathbf{v}_t);$$
$$\rho_t \partial U_t/\partial t + \rho_t(\mathbf{v}_t \cdot \nabla)\,U_t = -p_t\,\mathrm{div}\,\mathbf{v}_t - J_t U_t - J_s(\mathbf{v}_s - \mathbf{v}_t)^2/2 -$$
$$- J_r(\mathbf{v}_r - \mathbf{v}_t)^2/2 - \eta\left(I_{rs}\langle v_{rs}^2\rangle + I_{st}\langle v_{st}^2\rangle + I_{rt}\langle v_{rt}^2\rangle\right)/4 - \Delta_{tt}$$

(where ρ_i, $\mathbf{v}_i$ are the density and the velocity of the corresponding components, and p_t is the pressure of the randomly moving particles). The intensity of mass transfer between components of the mixture J_i is determined in accordance with the assumed rule for colliding particles entering type t:

$$J_t = -J_s - J_r,\ J_s = -I_{sr} - I_{st},\ J_r = -I_{rs} - I_{rt},$$
$$I_{ij} = 6\rho_i\rho_j\langle v_{ij}\rangle/(\rho_0 d_0).$$

Here $\langle v_{ij}\rangle$ is the mean magnitude of relative velocity of the colliding particles; and Δ_{tt} is the dissipation of energy of random motion of t particles due to incomplete elasticity of collisions between them, evaluated from the formula

$$\Delta_{tt} = 32\rho_t^2\eta U^{3/2}/(\sqrt{6\pi}\, d_0\rho_0),$$

where η is a coefficient characterizing the elasticity of the particles ($\eta = 0$ is absolutely elastic, and $\eta = 1$ is absolutely inelastic). The terms with the factor η in the last equation describe the conversion of kinetic energy of the particles into heat, and $\langle v_{ij}^2\rangle$ is the mean square relative velocity of colliding particles.

The pressure of the gas of t particles is found from the equation of state

$$p_t = (\varkappa_t - 1)\rho_t U_t$$

($\varkappa_t = 5/3$ for nonrotating particles or $\varkappa_t = 4/3$, if we account for their random rotation). To determine $\langle v_{ij}\rangle$ and $\langle v_{ij}^2\rangle$ we use kinetic theory formulas as for a gas of solid spheres, and η is found empirically. Equations for $\mathbf{v}_s$ and $\mathbf{v}_r$ are not required, since the velocities of the s and r particles along the trajectory do not change in the conditions examined.

Using this model we computed flow of solid particles over a sphere. The law of particle reflection from the surface was assumed to be specular (this corresponds to the assumption of an inviscid gas of t particles), and here the s particles transfer to type r, and for the t-particle gas the impermeability condition holds at the surface.

If we write the reduced equations in dimensionless form, referring the particle density to $\rho_{s\infty}$ (the subscript ∞ denotes parameters at infinity), the velocities to $u_{s\infty}$ (u_s is the component of $\mathbf{v}_s$ on the Ox axis, which is directed along the symmetry axis), the pressure to $\rho_{s\infty}u_{s\infty}^2$, and we take the sphere radius as the characteristic dimension (here the form of the equations is retained), then the problem will depend only on η, $\varkappa_t$, and $\mathrm{Kn} = d_0/(6\alpha_{s\infty})$, where $\alpha_{s\infty} = \rho_{s\infty}/\rho_0$ is the volume density of s particles. For example, the collision terms will have the form

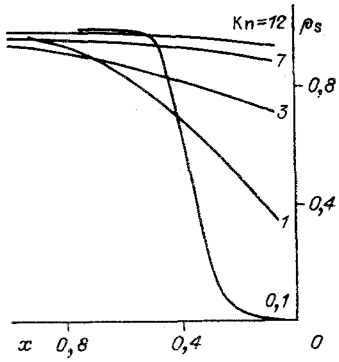

Fig. 1

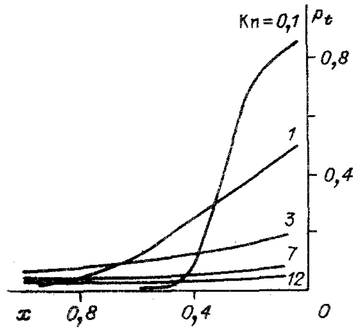

Fig. 2

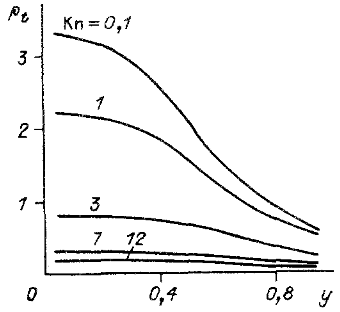

Fig. 3

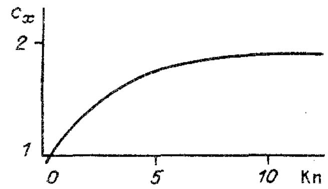

Fig. 4

$$I_{st} = \rho_s \rho_t \langle v_{st} \rangle / \mathrm{Kn},\ I_{rt} = \rho_r \rho_t \langle v_{rt} \rangle / \mathrm{Kn},$$
$$\Delta_{tt} = 16 \eta \rho_t^2 U^{3/2} / (3 \sqrt{6\pi}\, \mathrm{Kn}).$$

Here Kn is the analog of the Knudsen number in rarefied gas dynamics. The case considered, where there is no random particle motion in the undisturbed stream ($\rho_{t\infty} = 0$) is analogous to limiting hypersonic flow ($M \to \infty$) of gas over a body, and therefore the analog of Mach number does not appear in the list of governing parameters.

The flow over a sphere was computed by the Godunov method [6]. Ten mesh cells were fit to the shock layer thickness, achieving an accuracy sufficient for the qualitative analysis performed below.

Figure 1 shows curves of the fall of the s-particle density as we draw near the sphere surface for various values of Kn (the Ox axis is directed from the stagnation point against the main flow). Here and below we assume $\eta = 0$, $\varkappa_t = 5/3$. The range of Kn examined corresponds to the transition regime from flow of particles fully random ahead of the body to flow of individual particles similar to the transition regime in rarefied gas dynamics. For large Kn the particle density of the main flow varies only a little as one approaches the body surface. But as Kn is reduced still further the s particles reach the surface, and this is associated with an increased frequency of collisions between particles. For example, for Kn = 0.1 the s particles practically do not reach the surface. This value can be considered as a limit, below which there is a flow regime with a dense screening layer of reflected particles ahead of the body. In such a gas of t particles the pressure near the body is greater than for Kn > 0.1 (Fig. 2). The zone of propagation of random particles is located immediately adjacent to the body, forming some kind of shock layer. With increase of Kn the frequency of particle collisions decreases, the shock layer becomes thicker, and the pressure of t particles near the surface drops.

Figure 3 shows distribution curves of t-particle density on the sphere surface from the stagnation point to the mid-section for corresponding values of Kn. For all the regimes there is typically a high density of t particles near the stagnation point.

Figure 4 shows a curve of dependence of the drag coefficient c_x on Kn, where c_x is computed from the total momentum of s and t particles. As Kn increases $c_x \to 2$, which corresponds to the case where all the s particles reach the surface without collision, and for Kn $\to$ 0 the result is close to $c_x = 0.88$, the value given by the modified Newtonian theory for $M \to \infty$.

Qualitatively flow of elastic particles over a sphere is close to flow of a rarefied gas over a sphere, and with the model adopted we can compute the flow for Kn corresponding to the transition regime. This gives a basis to expect that the three-component model presented can be used to compute approximately the flow of a rarefied gas over bodies in the transition regime. To do this we need to use more realistic laws for the interaction of particles with the surface and to compute the viscosity of the t component. In addition, in order to be able to vary the Mach number we must introduce random motion of particles in the unperturbed flow, since in the model presented the flow of s particles corresponds to the limiting hypersonic case of $M \to \infty$.

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NONSELF-SIMILAR JET OF A NON-NEWTONIAN LIQUID

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The results of an analysis of the propagation of a two-dimensional submerged jet of a non-Newtonian liquid over the entire zone of its development are given within the framework of the boundary theory.

Jet flow is encountered in many technological applications. The pressing problem of analyzing non-Newtonian jet flow is created, in particular, by the broadening of the scope of application of polymers. Moreover, one must not forget the analogy between a turbulent flow and a non-Newtonian liquid with changes in the integral hydrodynamic parameters.

A self-similar solution was obtained earlier for a two-dimensional jet of a non-Newtonian liquid [1]. We shall investigate here the development of a two-dimensional jet of a non-Newtonian liquid throughout the entire region of its propagation by means of numerical calculations, using the method of local similarity. The Ostwald—de Ville model is used for approximating the flow rheology. Practical application of this model is justified in many cases of actual flow, for instance, polymer flow.

The initial equations of momentum transport and continuity of the submerged two-dimensional jet of a non-Newtonian liquid are given by

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